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Resonant frequencies in a container with a vertical baffle

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The effects of a vertical baffle on the resonant frequencies of fluid within a rectangular container are investigated using the linearized theory of water waves. The accuracy of simple approximate solutions is assessed by comparison with an accurate solution based on eigenfunction expansions. It is found that a surface-piercing barrier can change the resonant frequencies significantly while the effect of a bottom-mounted barrier is usually negligible.

1. Introduction

The resonant frequencies of oscillation of a liquid in a partly filled horizontal rectangular container are easily determined on the basis of linearized wave-wave theory. If a thin vertical baffle is introduced into the liquid so as to form two separate containers two sets of resonant frequencies exist appropriate to the dimensions of the respective containers. If the baffle is introduced only partly into the liquid it can be shown (Courant & Hilbert 1953) that the resonant frequencies are decreased in general, cxcept when the position of the baffle coincides with an antinode of an oscillation where the horizontal velocity is zero throughout the depth and the resonant frequency remains the same. As the baffle is introduced further and further into the fluid the nth resonant frequency changes continuously from its corresponding value in the absence of the baffle to the closest eigenfrequency not greater than it corresponding to the two separate containers.

The purpose of this paper is to examine precisely how the eigenfrequencies change with the position and depth of the baffle and the dimensions of the container.

In §2 the problem is formulated on the basis of linear water-wave theory. For simplicity, motion in the container is assumed to be restricted to be in vertical planes normal to the vertical plane of the baffle and is thus two-dimensional. By matching eigenfunction expansions valid either side of the baffle across the gap in the fluid not covered by the baffle, an integral equation is obtained for the unknown velocity across the gap, and an explicit condition derived for the resonant wavenumbers in terms of a quantity A related to this velocity. A similar approach has been used by others, notably Miles (1967, 1981) in related problems. The expansion of the unknown velocity in a series of orthogonal functions enables, after truncating the series, an explicit form to be derived for A as the ratio of two known determinants as described in Collin (1960) and used by Mei & Black (1969).

In §3 alternative approximate methods are described which avoid the numerical difficulties associated with the accurate approach. These include a one-term variational approximation, a 'wide-spacing' approximation, and a 'narrow-gap' approximation. Results for a range of parameters are presented in §4 where the relative merits of the approximations are discussed.

2. Formulation

Cartesian coordinates are chosen with $y = 0$ the undisturbed free surface. The walls of the tank are at $x = b, -c, 0 \leqslant y \leqslant h$ so that the tank is of uniform depth h. The baffle occupies the interval L', which is $x = 0, 0 \leqslant y \leqslant a$ for the surface-piercing baffle, and $x = 0, a \leqslant y \leqslant h$ for the bottom-mounted baffle, with $0 \leqslant a \leqslant h$ in each case. Most of what follows is easily adapted to more general situations in which there is a baffle with one or more gaps in the interval $x = 0, 0 \leqslant y \leqslant h$.

The usual assumptions of linearized water–wave theory ensure the existence of a velocity potential $\Phi(x, y, t)$. For simple harmonic motions we write $\Phi = \mathrm{Re}\,\phi(x, y)\exp(-\mathrm{i}\omega t)$, and then the time-independent potential $\phi(x, y)$ satisfies

$$\nabla^2\phi = 0 \quad \text{in the fluid,} \tag{2.1}$$

$$\kappa\phi + \frac{\partial\phi}{\partial y} = 0 \quad (y = 0, \quad -c < x < b, \quad x \neq 0), \tag{2.2}$$

where $\kappa = \omega^2/g$,

$$\frac{\partial\phi}{\partial y} = 0 \quad \text{on } y = h, \quad -c \leqslant x \leqslant b, \tag{2.3}$$

$$\frac{\partial\phi}{\partial x} = 0 \quad \text{on } L' \tag{2.4}$$

$$\frac{\partial\phi}{\partial x} = 0 \quad \text{on } x = b, -c, \quad 0 < y < h. \tag{2.5}$$

The eigenvalue problem defined by (2.1)–(2.5) is to be solved for allowable values of κ and hence ω. In the absence of the baffle,

$$\phi(x, y) = \cos k(b-x)\cosh k(h-y), \tag{2.6}$$

where

$$k = n\pi/d, \quad d = b+c \quad (n = 1, 2, \ldots), \tag{2.7}$$

are solutions. The resonant frequencies are then given by

$$\omega^2/g \equiv \kappa = k\tanh kh. \tag{2.8}$$

Similarly if the baffle extends throughout the depth, then there are two distinct sets of solutions

$$\phi(x, y) = \cos k(b-x)\cosh k(h-y) \quad (0 < x < b), \tag{2.9}$$

where

$$k = n\pi/b \quad (n = 1, 2, \ldots), \tag{2.10}$$

and

$$\phi(x, y) = \cos k(c+x)\cosh k(h-y) \quad (-c < x < 0), \tag{2.11}$$

where

$$k = m\pi/c \quad (m = 1, 2, \ldots). \tag{2.12}$$

The aim of the paper is to examine how these resonant frequencies are influenced by the presence of the baffle. In particular what is the dependence of kd on a/d, b/d and h/d, non-dimensional parameters describing the relative length and position of the baffle, and the tank aspect ratio.

We introduce the orthonormal eigenfunctions

$$\psi_n(y) = N_n^{-1} \cos k_n(h-y), \tag{2.13}$$

where k_n ($n = 1, 2, \ldots$) are the real positive roots of

$$\kappa + k_n \tan k_n h = 0, \tag{2.14}$$

and $k_0 = \mathrm{i}k$, and

$$2N_n^2 = \left(h + \frac{\sin 2k_n h}{2k_n}\right). \tag{2.15}$$

Then it follows that

$$\int_0^h \psi_n(y)\, \psi_m(y)\, \mathrm{d}y = \delta_{nm}. \tag{2.16}$$

We pose solutions in the form

$$\phi(x, y) = -\sum_{n=0}^{\infty} U_n \frac{\cosh k_n(b-x)}{k_n \sinh k_n b} \psi_n(y) \quad (0 \leqslant x \leqslant b), \tag{2.17}$$

$$= \sum_{n=0}^{\infty} U_n \frac{\cosh k_n(c+x)}{k_n \sinh k_n c} \psi_n(y) \quad (-c \leqslant x \leqslant 0), \tag{2.18}$$

where the U_n are the Fourier coefficients in the expansion of the horizontal velocity $U(y)$ across $x = 0$, $0 \leqslant y \leqslant h$. Thus

$$U(y) = \sum_{n=0}^{\infty} U_n \psi_n(y) \quad \text{with } U_n = \langle U, \psi_n \rangle \equiv \int_L U(y)\, \psi_n(y)\, \mathrm{d}y, \tag{2.19}$$

where L is the interval on $x = 0$ not occupied by the baffle, and we have used the condition $U(y) = 0$ on L'.

The forms (2.17), (2.18) satisfy conditions (2.1), (2.2), (2.3), (2.5) and ensure that the horizontal velocity $U(y)$ is continuous across L. Continuity of the dynamic pressure and hence the vertical velocity across L requires

$$\sum_{n=0}^{\infty} U_n k_n^{-1} (\coth k_n b + \coth k_n c)\, \psi_n(y) = 0 \quad (y \in L). \tag{2.20}$$

It follows on substitution of (2.19) that

$$\int_L U(t)\, K(y, t)\, \mathrm{d}t = 0 \quad (y \in L), \tag{2.21}$$

where

$$K(y, t) = \sum_{n=0}^{\infty} s_n\, \psi_n(y)\, \psi_n(t), \tag{2.22}$$

and

$$s_n = k k_n^{-1} (\coth k_n b + \coth k_n c), \tag{2.23}$$

and a factor k has been included for later convenience.

Now

$$K(y, t) = -A^{-1} \psi_0(y)\, \psi_0(t) + K_1(y, t), \tag{2.24}$$

where $K_1(y, t)$ is given by (2.22) without the term $n = 0$, and

$$A = (\cot kb + \cot kc)^{-1} = \frac{\sin kb \sin kc}{\sin kd}. \tag{2.25}$$

Thus, defining $u(y)$ by

$$U(y) = U_0 A^{-1} u(y) \tag{2.26}$$

(2.21) becomes

$$\int_L u(t)\, K_1(y, t)\, \mathrm{d}t = \psi_0(y) \quad (y \in L), \tag{2.27}$$

whilst multiplication of (2.26) by $\psi_0(y)$ and integration over L gives

$$\langle u, \psi_0 \rangle \equiv \int_L u(t)\, \psi_0(t)\, \mathrm{d}t = A. \tag{2.28}$$

It follows from (2.27) and (2.28) that

$$A = \langle u, \psi_0 \rangle^2 \Big/ \sum_{n=1}^{\infty} \langle u, \psi_n \rangle^2 s_n, \tag{2.29}$$

a form which can be shown to be stationary with respect to first-order variations of $u(y)$ about its true value, and that the resulting approximation is never greater than the true value of A.

Now $u(y)$ can be expanded in an infinite series of terms of the orthonormal set $\{\psi_m(y)\}$ $(m = 0, 1, 2, \ldots)$. If we substitute, as a trial function, the truncated expansion

$$u(y) = \sum_{m=0}^{M} u_m \psi_m(y) \tag{2.30}$$

into (2.29) we obtain

$$A = \frac{\boldsymbol{u}^{\mathrm{T}} \boldsymbol{C} \boldsymbol{u}}{\boldsymbol{u}^{\mathrm{T}} \boldsymbol{B} \boldsymbol{u}}, \tag{2.31}$$

where

$$\boldsymbol{u}^{\mathrm{T}} = (u_0, u_1, \ldots, u_m), \quad \boldsymbol{C} = \boldsymbol{c}\boldsymbol{c}^{\mathrm{T}}, \tag{2.32}$$

$$\boldsymbol{c}^{\mathrm{T}} = (c_{00}, c_{10}, \ldots, c_{m0}), \tag{2.33}$$

where

$$c_{mn} = \langle \psi_m, \psi_n \rangle \equiv \int_L \psi_m(y)\, \psi_n(y)\, \mathrm{d}y, \tag{2.34}$$

and

$$\boldsymbol{B} = \{B_{mn}\}$$

where

$$B_{mp} = \sum_{n=1}^{\infty} c_{mn} c_{pn} s_n. \tag{2.35}$$

The best possible approximation of the form (2.30) is now obtained by requiring the u_m to leave (2.29) stationary. The required condition is found, by differentiation of (2.31) with respect to the elements of $\boldsymbol{u}$, to be

$$\det(\boldsymbol{C} - A\boldsymbol{B}) = 0. \tag{2.36}$$

The form of $\boldsymbol{C}$ permits this to be solved for A as follows (Collin 1960 p. 359). Divide the mth row by c_{m0}, the nth column by c_{n0}, subtract the first row from the rest and factorize the resulting determinant. Then

$$A = \begin{vmatrix} 1 \quad 1 \cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots \dfrac{B_{0n}}{c_{0n}} - \dfrac{B_{mn}}{c_{mn}} \cdots \\ \cdots\cdots\cdots\cdots \end{vmatrix} \Bigg/ \begin{vmatrix} \cdots\, B_{0n}/C_{0n} \cdots \\ \cdots\cdots\cdots\cdots \\ \cdots \dfrac{B_{0n}}{c_{0n}} - \dfrac{B_{mn}}{c_{mn}} \cdots \\ \cdots\cdots\cdots\cdots \end{vmatrix}, \tag{2.37}$$

an explicit expression for A which can be made as accurate as required by increasing M (but see §4). The expression (2.37) in conjunction with (2.25) illustrates clearly the influence of the baffle on the eigenfrequencies. Thus it can be shown that A varies from infinity to zero as the baffle is lowered into the fluid until it totally separates the fluid region.

3. Approximate methods of solution

The computation of (2.36) or the direct determination of A from (2.37) both involve appreciable numerical work so it is desirable to seek simple approximate solutions.

3.1. *One-term variational approximation*

The first and simplest is to choose $M = 0$ in (2.33) or (2.34) to obtain a one-term variational approximation A_1 to the true solution, with $A_1 \leqslant A$, namely

$$A_1 = \left(\sum_{n=1}^{\infty} s_n c_{0n}^2\right)^{-1}. \tag{3.1}$$

It can be shown from (2.25) that the resulting values of kd are also lower bounds to the true values.

3.2. *The wide-spacing approximation*

For the higher modes at least, the endwalls of the tank will be many wavelengths from the baffle and it is possible to consider a wide-spacing approximation used to good effect in other water-wave problems by Ohkusu (1974), Newman (1977) and Srokosz & Evans (1979) and by Martin (1984) who gives a detailed discussion of the approximation involved.

The essential idea is that the wavefield away from the baffle may be approximated by a superposition of plane waves travelling in opposite directions, whilst any local evanescent modes are considered negligible. In the vicinity of the baffle it is assumed that the interaction of the wavefield with the baffle is governed by the appropriate reflection and transmission coefficients for waves incident upon the baffle in a fluid having a free surface extending to infinity in either direction.

Thus the velocity potential for the motion near $x = +b$ may be written

$$\left.\begin{aligned} \phi(x, y) &\approx 2B \cos k(x-b), \\ &= B(\mathrm{e}^{\mathrm{i}k(x-b)} + \mathrm{e}^{-\mathrm{i}k(x-b)}), \end{aligned}\right\} \tag{3.2}$$

whilst near $x = -c$,

$$\left.\begin{aligned} \phi(x, y) &\approx 2C \cos k(x+c), \\ &= C\,(\mathrm{e}^{\mathrm{i}k(x+c)} + \mathrm{e}^{-\mathrm{i}k(x+c)}). \end{aligned}\right\} \tag{3.3}$$

Here B, C are complex constants.

Each of these potentials consists of waves travelling both towards and away from the baffle. Now a wave travelling away from the baffle arises from the reflection of a wave in the same region and the transmission of a wave from the region on the other side of the baffle. It follows that

$$B\,\mathrm{e}^{-\mathrm{i}kb} = BR_2\,\mathrm{e}^{\mathrm{i}kb} + CT\,\mathrm{e}^{\mathrm{i}kc}, \tag{3.4}$$

and

$$C\,\mathrm{e}^{-\mathrm{i}kc} = CR_1\,\mathrm{e}^{\mathrm{i}kc} + BT\,\mathrm{e}^{\mathrm{i}kb}. \tag{3.5}$$

Here $R_1(R_2)$ is the reflection coefficient for waves incident upon the baffle from the left (right), whilst T is the transmission coefficient, known to be independent of the direction of the incident wave (Newman 1976). The condition for a solution of (3.4) and (3.5) to exist is

$$T^2\,\mathrm{e}^{\mathrm{i}kd} = (\mathrm{e}^{-\mathrm{i}kb} - R_2\,\mathrm{e}^{\mathrm{i}kb})\,(\mathrm{e}^{-\mathrm{i}kc} - R_1\,\mathrm{e}^{\mathrm{i}kc}). \tag{3.6}$$

This is a general expression derived under the wide-spacing approximation for the determination of the resonant frequencies in a tank containing any obstacle whatsoever, in terms of the reflection and transmission coefficients for that obstacle in an infinite wavetrain. For the particular problem being considered here, the obstacle is a thin vertical barrier or barriers on the line $x = 0$. It is known that for all such problems, $R_1 = R_2$ ($\equiv R$, say) and $R+T = 1$ enabling (3.6) to be reduced to

$$A \equiv \frac{\sin kb \sin kc}{\sin kd} = \frac{1-R}{2\mathrm{i}R} = \frac{T}{2\mathrm{i}R}. \tag{3.7}$$

It is straightforward to extend the method to more than one baffle but with a corresponding increase in algebraic complexity. Of particular interest to naval architects is the case of three identical baffles dividing the tank into four sections of equal length. Numerical computations of the forced motion in this type of tank are reported by Mikelis & Robinson (1985). If the four subsections of the tank are each of length b then the symmetric modes are given by (3.7) with $d = 2b$ and $c = b$. The antisymmetric modes are at wavenumbers satisfying

$$\tan kb = \frac{2r \pm (2r^2+1)^{\frac{1}{2}}}{2r^2-1}, \tag{3.8}$$

where $r = \mathrm{i}R/(1-R)$.

3.3. *The narrow-gap approximation*

If the surface-piercing baffle almost touches the bottom of the tank or the bottom-mounted baffle almost breaks the surface it is possible to use a narrow-gap approximation. The method which has been used with considerable success in water-wave problems was first used by Tuck (1971) in considering wave transmission through a narrow gap in a single vertical barrier. The idea is to match a local solution valid in the neighbourhood of the gap, where wave effects are not important to an 'outer' solution valid at some distance from the gap, in an overlap region, thereby communicating information on the details of the flow in the 'inner' region to the wave-like flow in the 'outer' region.

In the present problem the 'inner' solution is solved by a straightforward use of conformal mapping techniques, while the 'outer' solution corresponds to a symmetrically placed pulsating wave source bounded by vertical rigid walls and a rigid bottom. This solution is most easily obtained by Fourier transforms. The technique is well known and only the results are given here.

For the surface-piercing baffle, we find

$$A^{-1} = 4kN_0^2\left\{S_1 - \frac{1}{\pi}\ln\left[\tfrac{1}{2}\pi\left(1-\frac{a}{h}\right)\right]\right\} \tag{3.9}$$

where

$$S_1 = \sum_{n=1}^{\infty}\left\{\frac{s_n}{4kN_n^2} - \frac{1}{n\pi}\right\}, \tag{3.10}$$

whilst for the bottom-mounted baffle,

$$A^{-1} = 4k \operatorname{sech}^2 kh\, N_0^2\left\{S_2 - \frac{1}{\pi}\ln\frac{\pi a}{2h}\right\}, \tag{3.11}$$

where

$$S_2 = \sum_{n=1}^{\infty}\left\{\frac{s_n \cos^2 k_n h}{4kN_n^2} - \frac{1}{n\pi}\right\}. \tag{3.12}$$

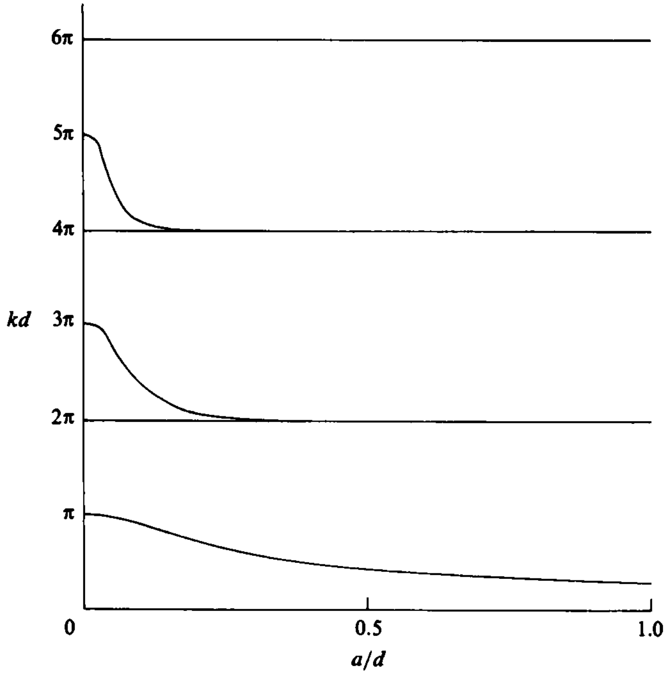

FIGURE 1. Wavenumber kd *vs.* baffle length a/d for surface-piercing baffle; $b/d = 0.5$, $h/d = 1$.

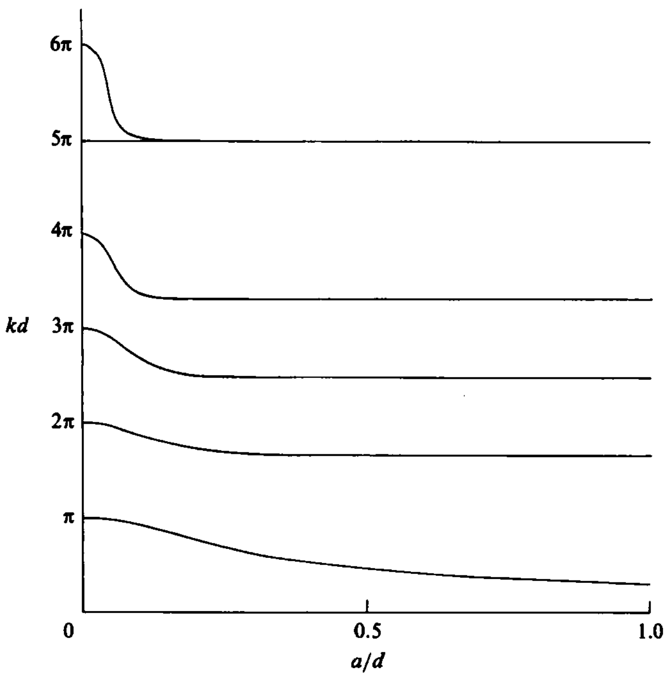

FIGURE 2. Wavenumber kd *vs.* baffle length a/d for surface-piercing baffle; $b/d = 0.4$, $h/d = 1$.

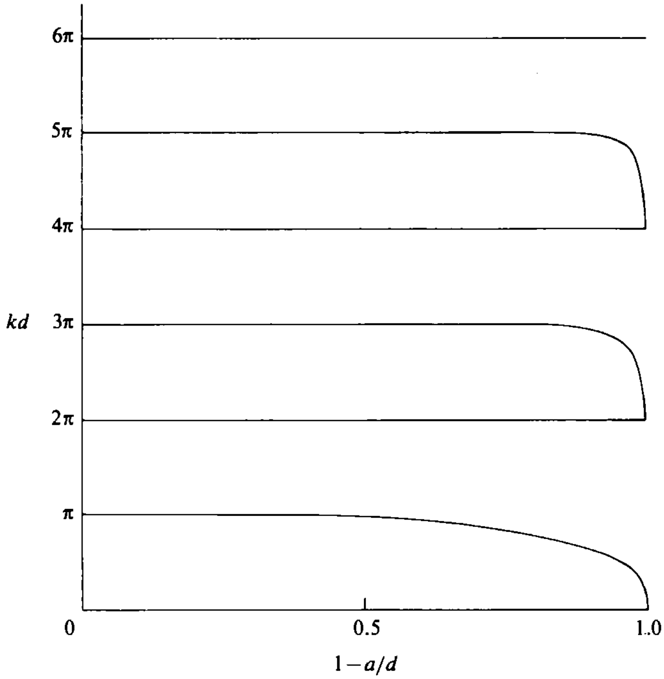

FIGURE 3. Wavenumber kd *vs.* baffle length $1-a/d$ for bottom-mounted baffle; $b/d = 0.5$, $h/d = 1$.

4. Results

4.1. *Full linear theory*

In order to determine A and hence kd using the full linear theory the matrices $\boldsymbol{C}$ and $\boldsymbol{B}$ must be computed. Whilst $\boldsymbol{C}$ is straightforward, the elements of $\boldsymbol{B}$ are infinite sums requiring truncation. A discussion of the factors influencing the level of truncation in a related problem is given by Evans & McIver (1984). In the present work it was found that satisfactory convergence for the element B_{mn} was obtained by summing to 10 (max $(m, n) + 100$) terms which is somewhat more terms than usually required in problems of this type.

Although A is given explicitly by (2.37) in terms of the elements of $\boldsymbol{B}$ and $\boldsymbol{C}$ it turns out that the determinants in (2.37) are highly ill-conditioned, with rapid variation of A occurring for small changes in the parameters. In contrast a standard root-finding routine enabled the value of kd to be found in a straightforward manner from (2.36) with A given by (2.25) and this method was used for all the linear theory calculations. The eigenvalue theory of Courant & Hilbert (1953) provided bounds which simplified the calculations. It was found that for $M = 5$, the value of kd obtained was accurate to two decimal places except when the tank was almost totally separated by the baffle. In all the results presented here M was chosen to be 20 which was usually sufficient for three-figure accuracy.

The results are presented as curves of kd against a/d for different values of b/d, and h/d. The resonant frequencies can then be determined from (2.8) if necessary.

Figure 1 shows results for a surface-piercing baffle placed centrally in the tank. The symmetric modes of the unobstructed tank ($kd = 2n\pi$, n integer) have zero horizontal velocity on the centreline and so are unaffected by the baffle. The value of kd for the

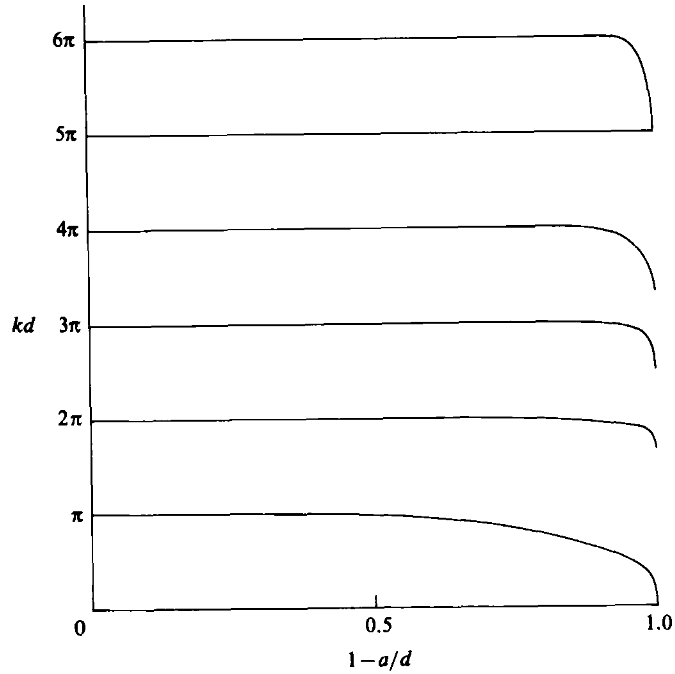

FIGURE 4. Wavenumber kd *vs.* baffle length $1-a/d$ for bottom-mounted baffle; $b/d = 0.4$, $h/d = 1$.

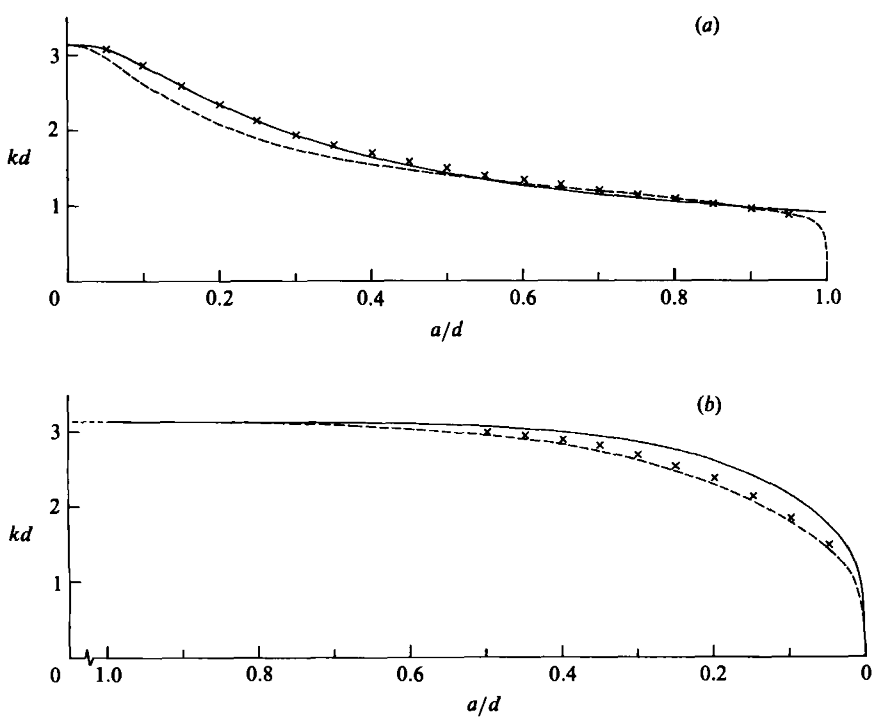

FIGURE 5. Comparison of wide-spacing (——) and one-term variational (---) approximations with the full theory (×). Lowest mode for (*a*) surface-piercing baffle, (*b*) bottom-mounted baffle; $b/d = 0.5$, $h/d = 1$.

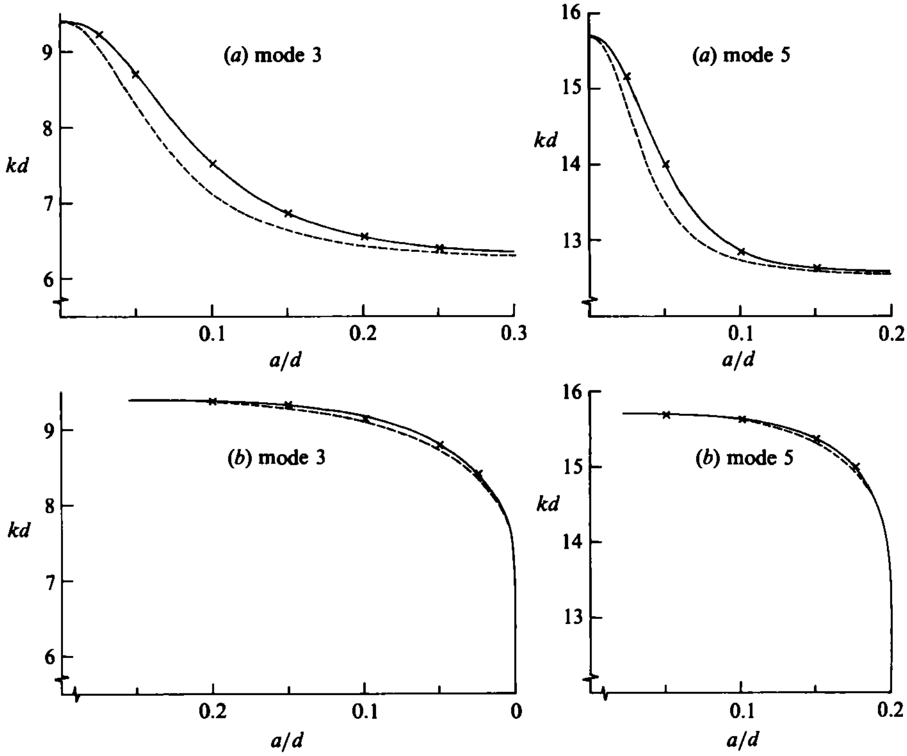

FIGURE 6. Comparison of wide-spacing (——) and one-term variational (---) approximations with the full theory (×). Modes three and five for (a) surface-piercing baffle, (b) bottom-mounted baffle; $b/d = 0.5$, $h/d = 1$.

lowest mode is slowly reduced as the submergence of the baffle is increased, until the tank is almost fully divided when there is a very rapid drop to zero. A similar phenomenon has been observed by Tuck (1980) when considering the effect of a submerged barrier on the natural frequencies of a basin connected to open water. As might have been anticipated, the higher-frequency modes are reduced to their lower limiting values much more rapidly as the submergence increases. When the baffle is not centrally placed, as in figure 2, some of the symmetric modes are affected by the presence of the baffle. Whenever the ratio of the widths of the basins on either side of the baffle is rational there will be some unaffected symmetric modes.

For the case of a bottom-mounted baffle the behaviour differs only in detail, as can be seen in figures 3 and 4. Note that a is here the submergence of the baffle tip and that the horizontal axis is reversed so that the fully-divided tank again lies on the right of the figures. For this geometry the barrier must extend over a substantial part of the depth before any significant change in the natural frequencies will occur. The higher modes are less affected by the baffle as might be expected on physical grounds.

4.2. *Approximate solutions*

Three different approximate expressions for A have been described from which, using (2.25), (2.8) the eigenfrequencies can be estimated. The least useful of these is the narrow-gap approximation given by (3.8)–(3.11). This gives very accurate results when compared with results from the full linear theory when the gap is of the order of a tenth of the depth or smaller but rapidly diverges from the full solution as the

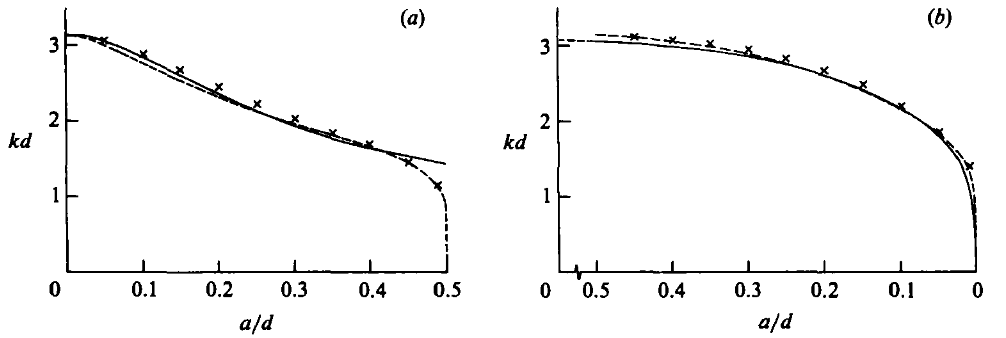

FIGURE 7. Comparison of wide-spacing (——) and one-term variational approximations (---) with full theory (×). Lowest mode for (*a*) surface-piercing baffle, (*b*) bottom-mounted baffle; $b/d = 0.5$, $h/d = 0.5$.

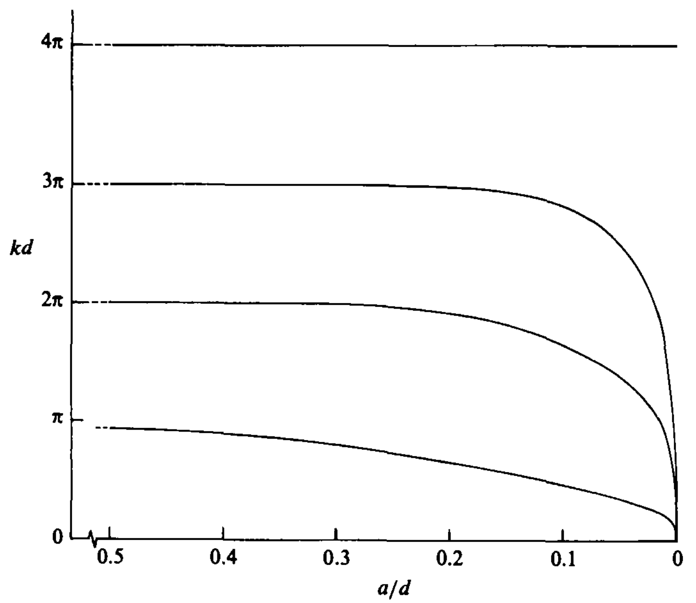

FIGURE 8. Wavenumber kd *vs.* baffle submergence a/d for a tank of width d containing three equally spaced bottom-mounted baffles.

gap is increased. The one-term variational approximation given by (3.1) performs almost as well for small gaps and also reproduces the general behaviour elsewhere.

For a surface-piercing baffle the wide-spacing approximation given by (3.6) with the deep-water reflection coefficient $R = \pi I_1(\kappa a)/(\pi I_1(\kappa a) + \mathrm{i}K_1(\kappa a))$, (Ursell 1947) is accurate for all modes and over a wide range of parameters as is indicated by the results of figure 5(*a*) and 6(*a*). (In figure 6(*a*) only that part of the curves where kd differs sensibly from its limiting value of $n\pi$ is shown.) This approximation only fails for the lowest mode in not giving the small-gap behaviour (figure 5*a*). The one-term variational approximation does give this behaviour in agreement with the narrow-gap approximation, although it is generally less accurate for larger gaps and for the higher modes.

For bottom-mounted baffles the corresponding deep water reflection coefficient $R = K_0(\kappa a)/(\mathrm{i}\pi I_0(\kappa a) + K_0(\kappa a))$ (Ursell 1947) is used in the wide-spacing approximation and it is seen from figure 5(*b*) that this is inferior to the one-term variational approximation for the lowest mode but, from figure 6(*b*) superior for the higher modes.

So far all the calculations reported have been for a depth to width ratio of unity. Figure 7 gives results for the lowest mode in a shallower tank where $h/d = 0.5$. For a surface-piercing baffle, the effects of the reduced depth do not become significant until the gap is quite small. Once again, except when the gap is small the wide-spacing approximation is superior to the variational approximation although the only way in which the depth of the tank enters the approximation is through (2.8). A bottom-mounted baffle is now more effective in reducing the value of kd, since it need extend over a smaller proportion of the depth to produce a given change in kd. Again it is clear from figure 7(*b*) that the variational approximation is best for the lowest mode. The loss of accuracy for shallow depths of the wide-spacing approximation may be seen to some extent in figure 7 (*b*) for $a/d = 0.5$ where kd should equal π. This 'short-fall' becomes more apparent as the depth is further reduced.

Finally results are given in figure 8 for a tank containing three identical bottom-mounted baffles. These calculations were made using the wide-spacing approximation (equations (3.7) and (3.8)) with the deep water reflection coefficient (given above). The lowest four modes are displayed, modes two and four are symmetric modes and correspond to the lowest two modes of a tank of width $2b$ containing one centrally placed baffle.

Conclusion

The effect of introducing a thin vertical baffle into a rectangular two-dimensional wave tank is to lower the eigenfrequencies. Roughly speaking, a half-immersed surface-piercing baffle reduces the lowest resonant wave-number to less than half. In contrast, a bottom-mounted baffle of the same length has negligible effect on the resonant wavenumbers. The method described here, using linear water-wave theory, enables the eigenfrequencies to be computed for either a bottom-mounted or surface piercing vertical baffle of any length and position and for all tank dimensions.

Of particular interest is the simple formula

$$\frac{\sin kb \sin kc}{\sin kd} = \frac{1-R}{2iR}, \tag{3.7}$$

for determining the eigenfrequencies in terms of the reflection coefficient for the baffle in an infinite wavetrain, based on the wide-spacing approximation. It has been shown that even using the infinitely deep-water expression for R (3.7) predicts the resonant frequencies accurately over a wide range of parameters.

The extension to a three-dimensional rectangular tank with a baffle extending the full width is straightforward although the approximate expression (3.7) requires the corresponding value of R for obliquely-incident waves which is not known explicitly.

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